Project: ***THE INSIDERS–OUTSIDERS SOCIETY: A THEORETICAL AND EMPIRICAL INVESTIGATION OF THE POLITICO-ECONOMIC SYSTEM***

supported by ELIDEK grant No 11347501

Work Package 3: A DSGE Model of the Politico-Economic Equilibrium of the Insiders-Outsiders Society: Growth and Business Cycle Properties and Estimation

Empirical estimation (WP3B)

July 5th, 2024

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Private Sector Equilibrium- description

- **NO external (international) sector**
- **NO public debt, NO government transfers**

- **Representative household with leisure-labor choice**

$$u_t^h = \frac{[c_t^{\vartheta}(1-h_t)^{1-\vartheta}]^{1-\gamma}}{1-\gamma}; \gamma > 0$$

and intertemporal maximization.

PSE- description

- **Two-sector Intermediate goods production (Cobb-Douglas) with**
  - **"Outsiders" sector with fully competitive industries**

$$y_t^o(\varsigma) = B_t^o k_t^o(\varsigma)^{\alpha} l_t^o(\varsigma)^{(1-\alpha)}; \alpha \in (0,1)$$

  - **"Insiders" sector with**
    - **monopolies under unionized wage bargaining**

$$y_t^i(\varsigma) = B_t^i k_t^i(\varsigma)^{\alpha} l_t^i(\varsigma)^{(1-\alpha)}$$

PSE- description

- **Single-sector (composite) final good production with C.E.S. production**

$$y_t = A_t\left[\varphi_1 \chi_t^{\varphi_2}\left(\mathrm{Y}_t^i\right)^{\phi} + (1-\varphi_1)(1-\chi_t)^{\varphi_2}\left(\mathrm{Y}_t^o\right)^{\phi}\right]^{(1/\phi)}$$

$$\mathrm{Y}_t^i = \left[\int_0^{\chi_t} y_t^i(\varsigma)^{\theta} d\varsigma\right]^{(1/\theta)} ;\theta \in (0,1)$$

$$\mathrm{Y}_t^o = \left[\int_{\chi_t}^{1} y_t^o(\varsigma)^{\theta} d\varsigma\right]^{(1/\theta)}$$

- **ALERT : we will eventually be using**

$$y_t = A_t\left[\frac{1}{2}\left(\mathrm{Y}_t^i\right)^{\phi} + \frac{1}{2}\left(\mathrm{Y}_t^o\right)^{\phi}\right]^{(1/\phi)}$$

PSE- description

- **Government** with balanced budget
  - **Collects** tax on labor and capital rewards
  - **Spends** on
    - "conventional" government expenses
    - Expenses to maintain/change the "Insiders Index"
    - Does NOT optimize ("PSE" equlibrium)

$$\chi_{t+1} \in (0,1)$$

- **The Insiders Index**

  Is the proportion of Insiders industries over all industries in the two sectors of intermediate goods.

PSE- solution

- **Familiar-looking equations with "wedges" representing the effects of the Insiders existence (the "omega" variables)**

$$\frac{1-\vartheta}{\vartheta}\frac{c_t}{1-h_t}=\breve{\omega}_t(\chi_t)(1-\tau_t^L)(1-\alpha)\frac{y_t}{h_t}$$

$$y_t=\bar{\omega}_t(\chi_t)k_t^{\alpha}h_t^{(1-\alpha)}$$

$$\bar{\psi}_t+\hat{\psi}\chi_t+\frac{1}{2}\hat{\psi}(\chi_{t+1}-\chi_t)^2=\tau_t^K\left[\alpha\bar{\omega}_t(\chi_t)-\delta\left(\frac{k_t}{y_t}\right)\right]+(1-\alpha)\tau_t^L\tilde{\omega}_t(\chi_t)$$

$$\frac{u_t}{c_t}=\frac{\beta}{(1+\eta)}E_t\left\{\frac{u_{t+1}}{c_{t+1}}\left[1+(1-\tau_{t+1}^K)\left[\bar{\omega}_{t+1}(\chi_{t+1})\alpha\frac{y_{t+1}}{k_{t+1}}-\delta\right]\right]\right\}$$

PSE- solution

- **Deciphering of the omega wedges**

$$\xi \equiv \frac{(1-\alpha)\theta + \left\{1-\alpha\theta-[(1-\alpha\theta)+(1-\theta)]\lambda\right\}\left(\frac{\mu}{1-\mu}\right)}{(1-\alpha)\left[1+(1-\lambda)\left(\frac{\mu}{1-\mu}\right)\right]}$$

$$\Delta_t(\chi_t) \equiv \left(\frac{\varphi_1}{1-\varphi_1}\right)^{1/(1-\phi)} \left[\theta^{\alpha}\xi^{(1-\alpha)}\right]^{\phi/(1-\phi)} \left(B_t\right)^{\phi/(1-\phi)} \left(\frac{\chi_t}{1-\chi_t}\right)^{\frac{\theta\varphi_2+\phi(1-\theta)}{\theta(1-\phi)}}, \qquad B_t \equiv \frac{B_t^i}{B_t^0}$$

$$\bar{\omega}_t(\chi_t) \equiv \frac{1+\theta\Delta_t(\chi_t)}{1+\Delta_t(\chi_t)} \qquad \breve{\omega}_t(\chi_t) \equiv \frac{1+\xi\Delta_t(\chi_t)}{1+\Delta_t(\chi_t)} \qquad \tilde{\omega}_t(\chi_t) \equiv \left[\frac{1+\left[(1-\alpha\theta)/(1-\alpha)\right]\Delta_t(\chi_t)}{1+\Delta_t(\chi_t)}\right]$$

$$\bar{\omega}_t(\chi_t) \equiv \tilde{A}_t \frac{(1-\varphi_1)^{(1/\phi)}(1-\chi_t)^{\frac{\theta\varphi_2+(1-\theta)\phi}{\theta\phi}}\left[1+\Delta_t(\chi_t)\right]^{(1/\phi)}}{\left[1+\theta\Delta_t(\chi_t)\right]^{\alpha}\left[1+\xi\Delta_t(\chi_t)\right]^{(1-\alpha)}}, \qquad \tilde{A}_t \equiv A_t B_t^o$$

Empirical state-space model

The model has unobservable state variables.

STATE-SPACE model / Log-linear-around-the-steady-state

$$\hat{z}_t \equiv \ln z_t - \ln \bar{z}$$

1a. Structural measurement equation (invertible)

$$\begin{bmatrix} \theta_{11} & -1 & \theta_{13} & 1 \\ \theta_{21} & 1 & 0 & 0 \\ 0 & \theta_{32} & 1 & 0 \\ \theta_{41} & \theta_{42} & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{h}_{t+1} \\ \hat{y}_{t+1} \\ \hat{\tau}^L_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} d_h \\ d_y \\ d_\tau \\ d_c \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \lambda_{41} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{h}_t \\ \hat{y}_t \\ \hat{\tau}^L_t \\ \hat{c}_t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ m_{21} & 0 \\ m_{31} & m_{32} \\ m_{41} & m_{42} \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\tau}^K_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & \xi_{12} & \xi_{13} & 0 \\ 1 & \xi_{22} & \xi_{23} & 0 \\ 0 & \xi_{32} & \xi_{33} & \xi_{34} \\ 0 & \xi_{42} & \xi_{43} & 0 \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \\ \ln \bar{\psi}_{t+1} \end{bmatrix}$$

Empirical state-space model

1b. Reduced-form measurement equation

$$\begin{bmatrix} \hat{h}_{t+1} \\ \hat{y}_{t+1} \\ \hat{\tau}^{L}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} c_h \\ c_y \\ c_\tau \\ c_c \end{bmatrix} + \begin{bmatrix} \gamma_{11} & 0 & 0 & \gamma_{14} \\ \gamma_{21} & 0 & 0 & \gamma_{24} \\ \gamma_{31} & 0 & 0 & \gamma_{34} \\ \gamma_{41} & 0 & 0 & \gamma_{44} \end{bmatrix} \begin{bmatrix} \hat{h}_t \\ \hat{y}_t \\ \hat{\tau}^{L}_t \\ \hat{c}_t \end{bmatrix}$$

$$+ \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \\ \psi_{31} & \psi_{32} \\ \psi_{41} & \psi_{42} \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\tau}^{K}_{t+1} \end{bmatrix} + \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{41} & \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \\ \ln \bar{\psi}_{t+1} \end{bmatrix} + \begin{bmatrix} \varepsilon^{h}_{t+1} \\ \varepsilon^{y}_{t+1} \\ \varepsilon^{\tau}_{t+1} \\ \varepsilon^{c}_{t+1} \end{bmatrix}$$

Empirical state-space model

2. Transition equation (unobservable state variables)

$$\begin{bmatrix} \ln \tilde{A}_{t+2} \\ \ln B_{t+2} \\ \ln \chi_{t+2} \\ \ln \bar{\psi}_{t+2} \end{bmatrix} = \begin{bmatrix} d_A \\ d_B \\ d_\chi \\ d_{\bar{\psi}} \end{bmatrix} + \begin{bmatrix} \rho^A & 0 & 0 & 0 \\ 0 & \rho^B & 0 & 0 \\ 0 & 0 & \rho^\chi & 0 \\ 0 & 0 & 0 & \rho^{\bar{\psi}} \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \\ \ln \bar{\psi}_{t+1} \end{bmatrix} + \begin{bmatrix} u^A_{t+1} \\ u^B_{t+1} \\ u^\chi_{t+1} \\ u^\psi_{t+1} \end{bmatrix}$$

$$d_A = \left(1-\rho^A\right)\ln \bar{\tilde{A}}, \quad d_B = \left(1-\rho^B\right)\ln \bar{B}\ , \quad d_\chi = \left(1-\rho^\chi\right)\ln \bar{\chi}, \quad d_{\bar{\psi}} = \left(1-\rho^{\bar{\psi}}\right)\ln \bar{\bar{\psi}}$$

Calibrated (never-estimated) Things

1. Steady-states of data series = time series mean value

2. Parameters

a/a	Symbol	Description	Values
1.	δ	capital depreciation rate	Calibrated **per country** in order to obtain the Capital time series
2.	η	growth rate of labor-augmenting technical progress	Calibrated **per country** to reflect trends in output
3.	ϑ	intensity of consumption in household preferences, relative to leisure. For well-behaved utility we need $0 < \vartheta < 1$	Calibrated **per country** so that the great ratios of the TM are close to the corresponding values implied by the data.
4.	γ	where $1/\gamma$ is the intertemporal elasticity of substitution of households. For bounded utility we need $\gamma > 1$	**2.00** common value
5.	φ_1	Final-good production function constant distribution parameter	**0.5** common value
6.	φ_2	Exponent of Insiders index in the final good production function	**0.0** common value

Empirical model – estimation goals

1. **Economic parameters to be estimated**

1.	α	capital elasticity of output in the intermediate Sectors
2.	θ	where $1/(1-\theta)$ is the aggregation elasticity of substitution between the outputs of intermediate industries when used as aggregate input in the production of the final good . In equilibrium, $\theta = \mathrm{APK}^o/\mathrm{APK}^i$ i.e. it is the ratio of average capital products of the intermediate sectors.
3.	ϕ	where $1/(1-\phi)$ is elasticity of substitution across sectors in final good production
4.	ξ	In equilibrium $\xi = \mathrm{APL}^o/\mathrm{APL}^i$, namely, it is the ratio of average labor products, of the intermediate sectors.
5.	$\hat{\psi}$	Rate of direct increase in government expenses as the Insiders Index changes. It multiplies χ_t

2. **Autocorrelation coefficients of unobservable states**
3. **Steady-states of unobservable states**
4. **Predicted Time-series of unobservable states**

DATA

- OECD data base
- Real, quarterly, seasonally adjusted
- Countries: France, Germany, Greece, Italy, Portugal, Spain, UK
- Range: 1999-q1 to 2018-q2

  except Germany : 2002-q1 to 2018-q3

- Output, consumption, capital expressed in Euros per "efficiency-augmented available-for-work" hour
- Effective labor and capital tax rates in proportions
- Net exports added to consumption

- Capital series constructed by perpetual Inventory method (using the period 1995-q1 to 1998-q4 to pin K_0)

PHASE 1 – predicting the unobservables

- **NO parameter estimation – they stay in their starting values**

  …that are set per country so as the theoretical model equations align with the data averages.

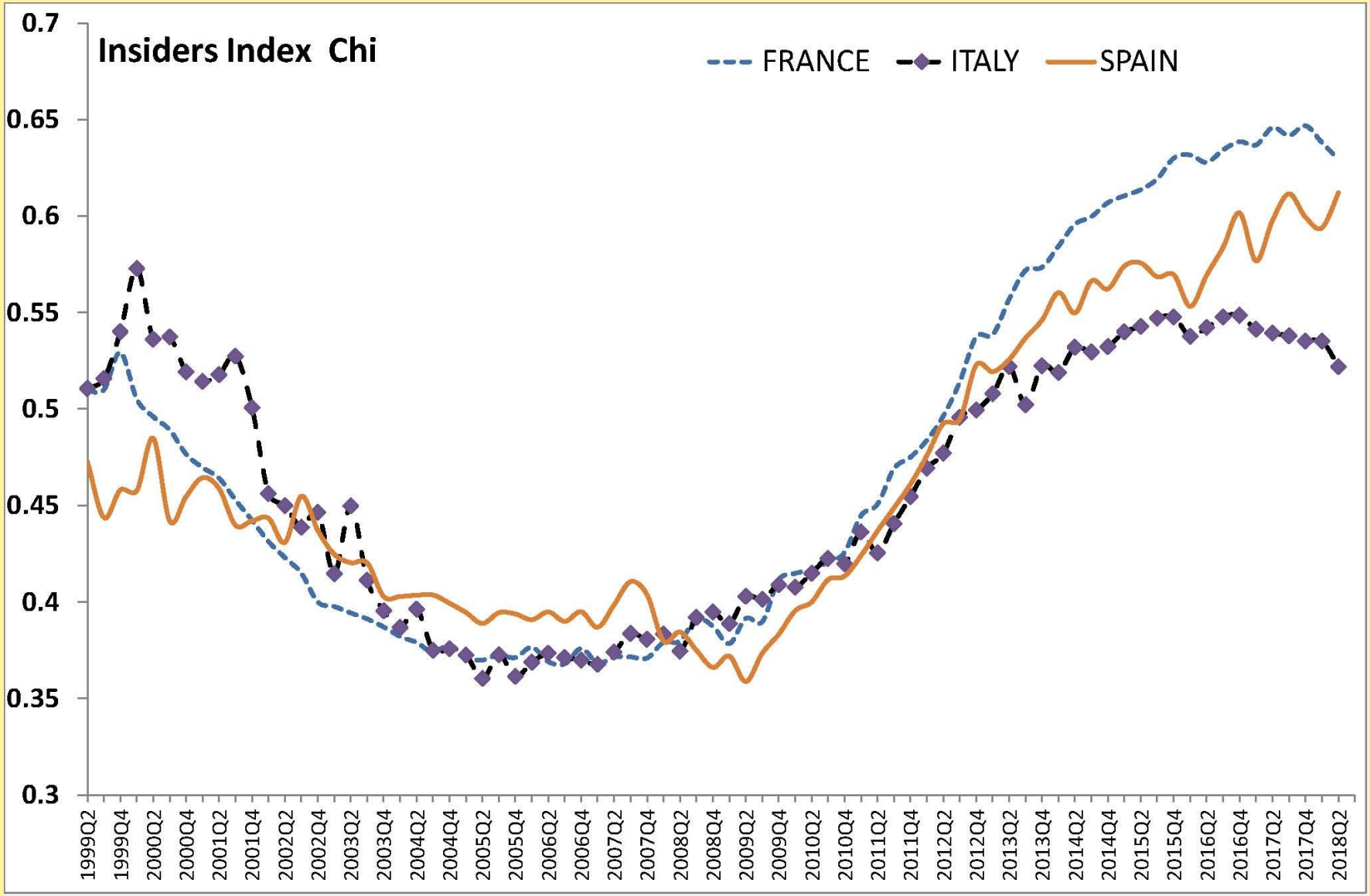
  - **Autocorrelation coefs of the states are estimated.**

- **Given them and the Data, the (smoothed) Kalman Filter provides predicted series for the unobservable state variables.**
- **Usefulness:**

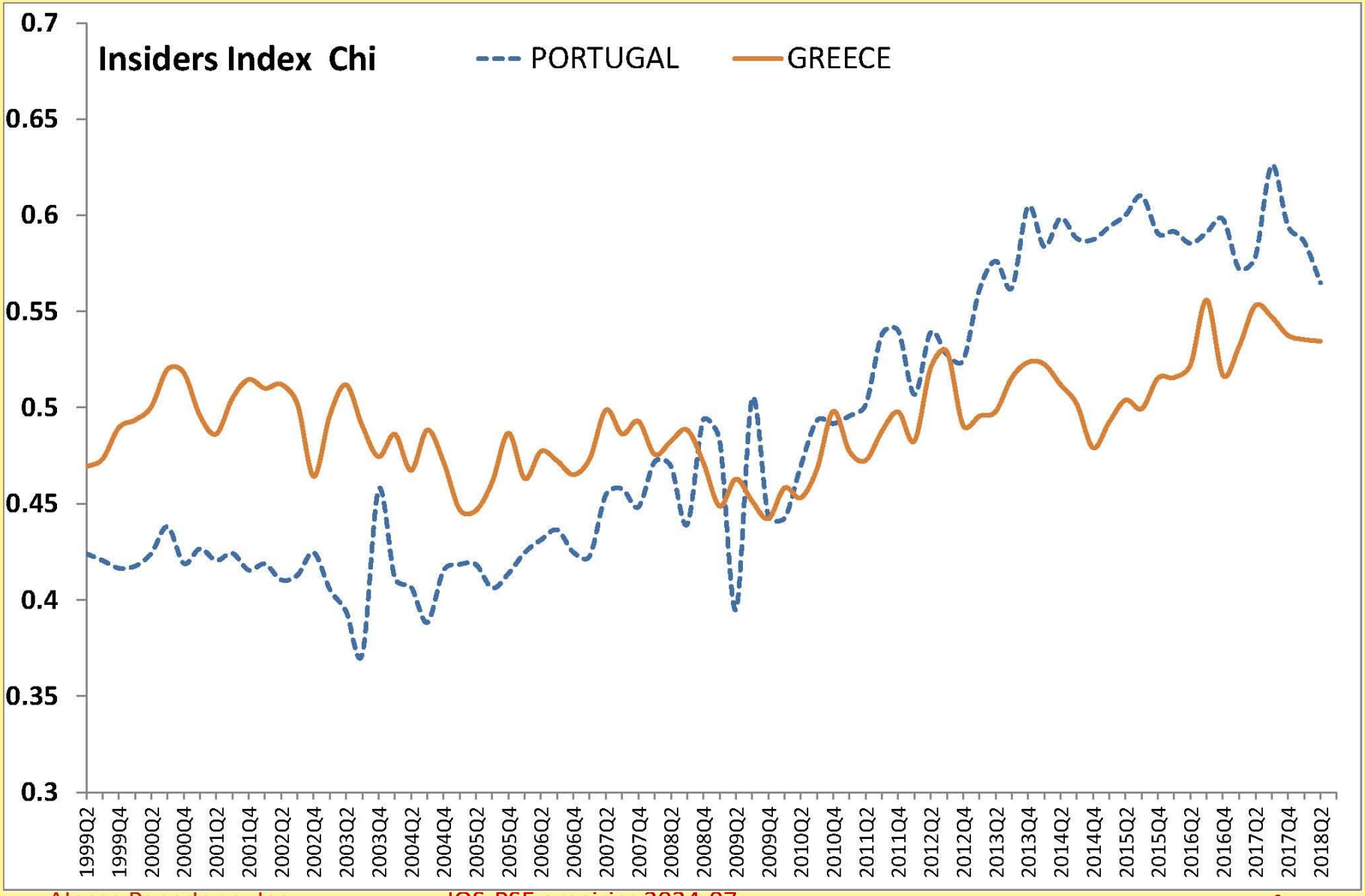
  1. This is not a calibration exercise. The model works with actual data. So we check broad compatibility and empirical relevance of the theoretical and of the empirical model.

  2. The results can be valid on their own, subject to robustness checks.

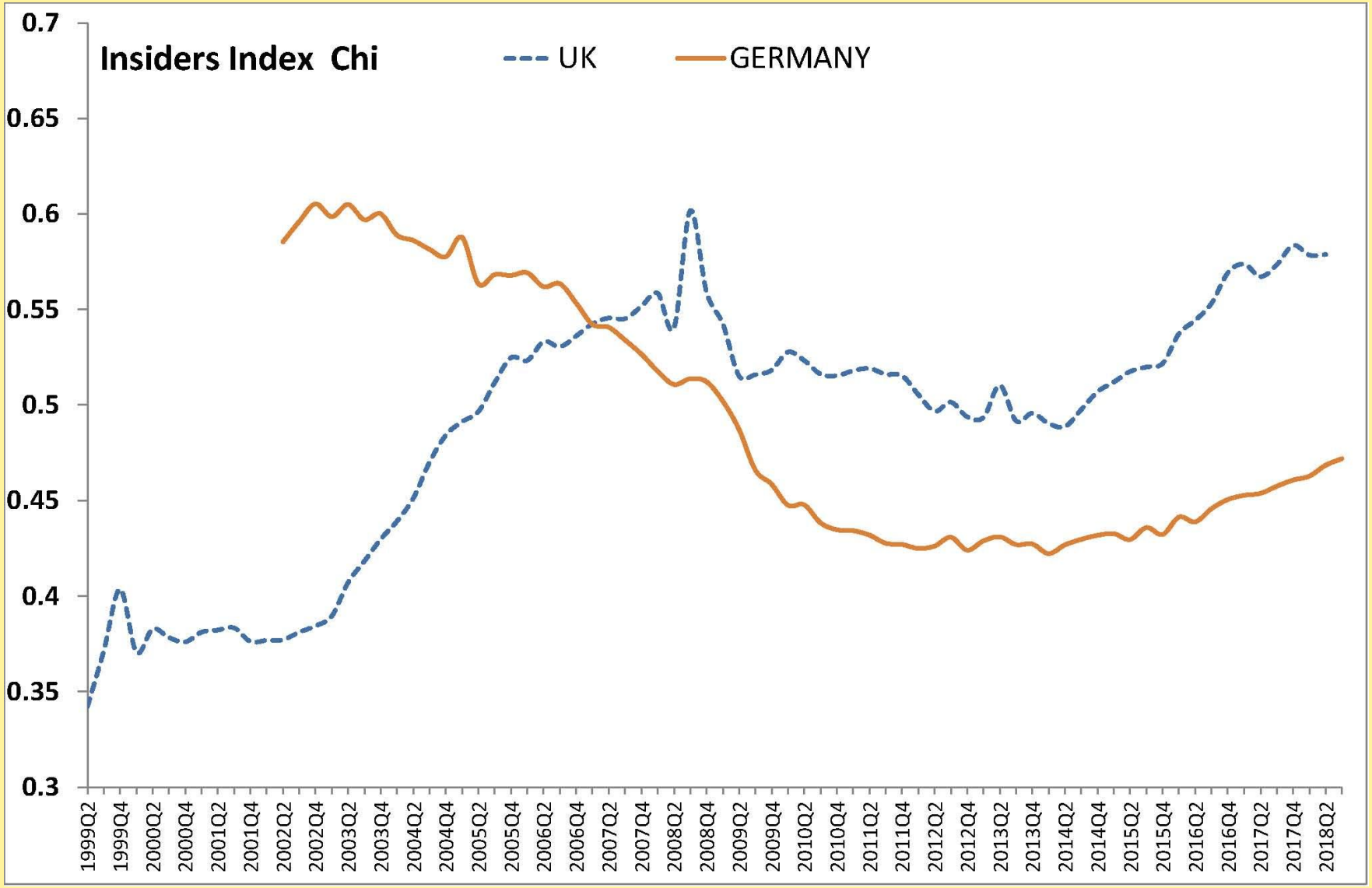
KF predicted Unobservables – Insiders Index



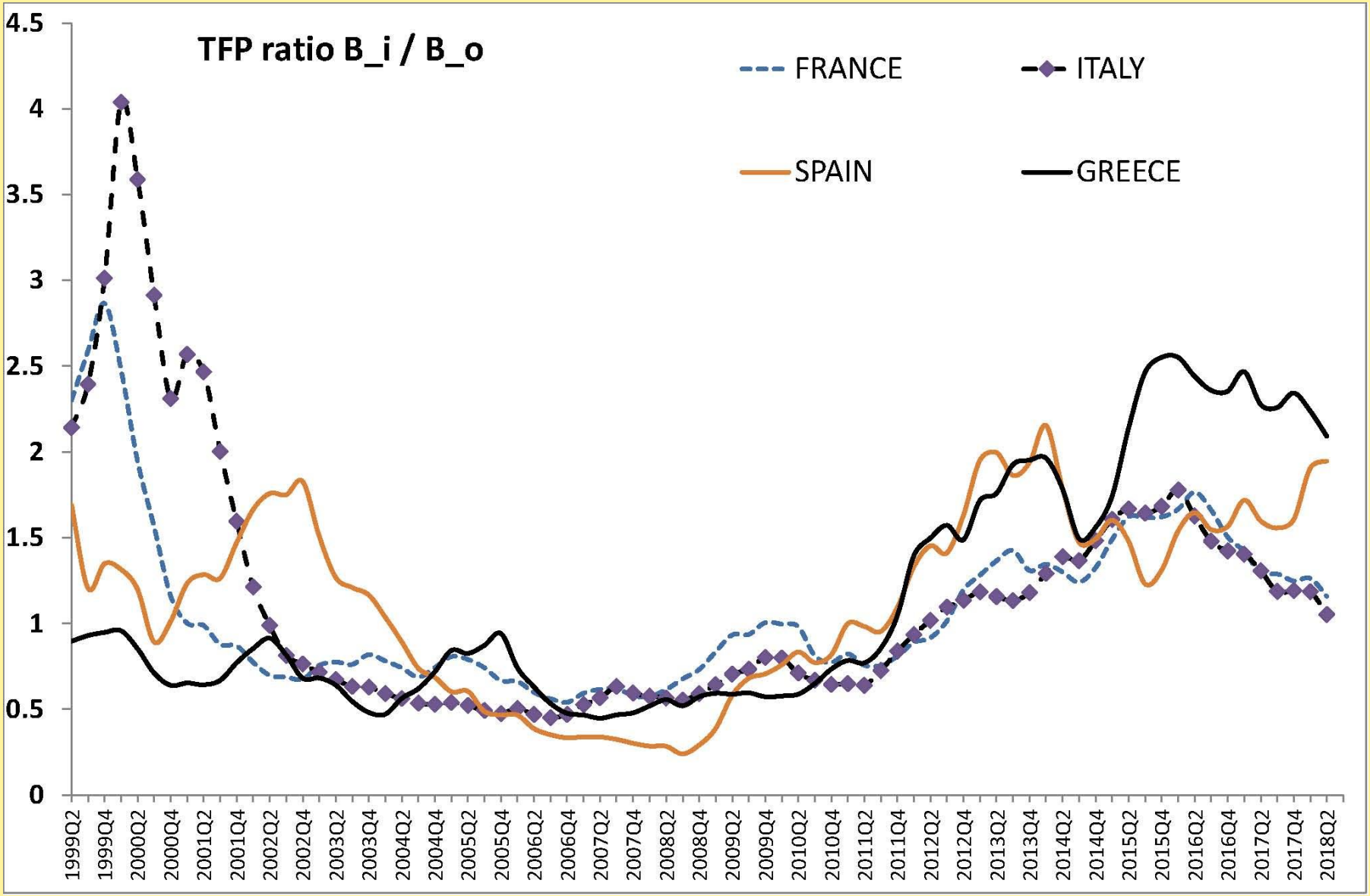
KF predicted Unobservables – Insiders Index



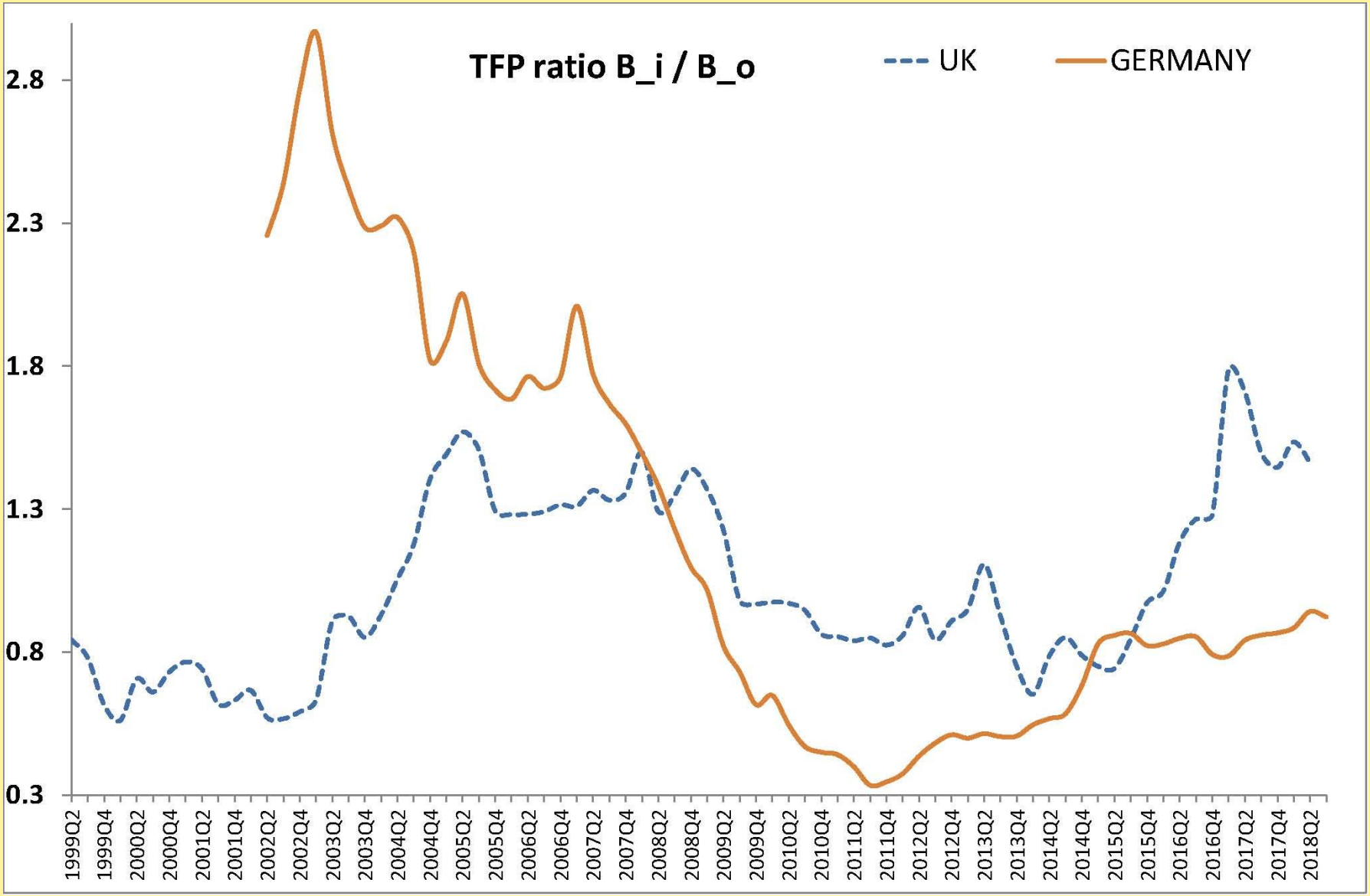
KF predicted Unobservables – Insiders Index



KF predicted Unobservables – TFP ratio



KF predicted Unobservables – TFP ratio



KF predicted Data

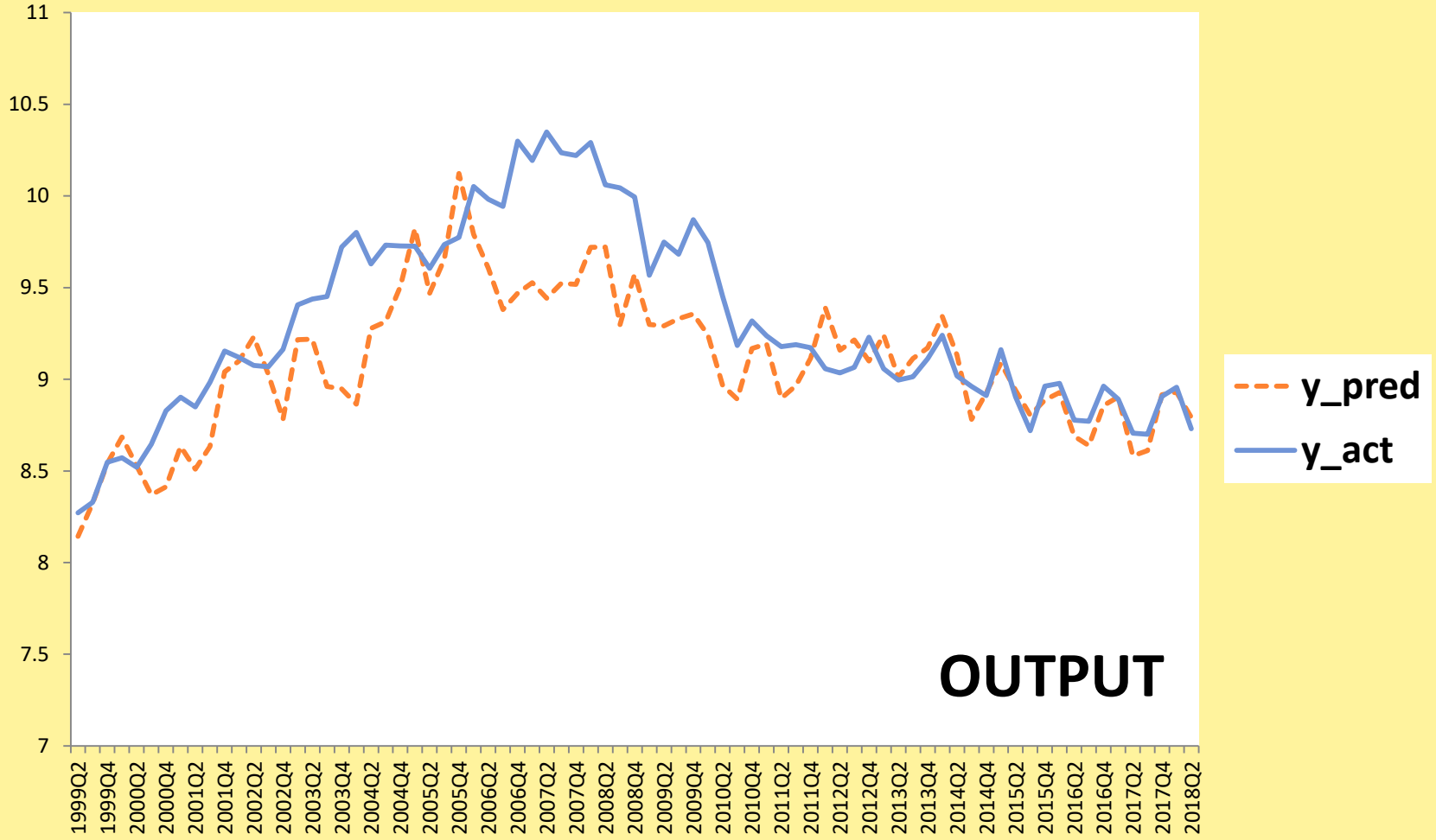
- **General Tendency: Underestimate the data series**

- **Acceptable prediction of Output**
- **Mostly Bad predictions of consumption and hours worked (*h, proportion of time worked)***

- **BUT**
  - **If we use the predicted series for output and**
  - **the not-good prediction of the one**
  - **We get a good prediction for the other**

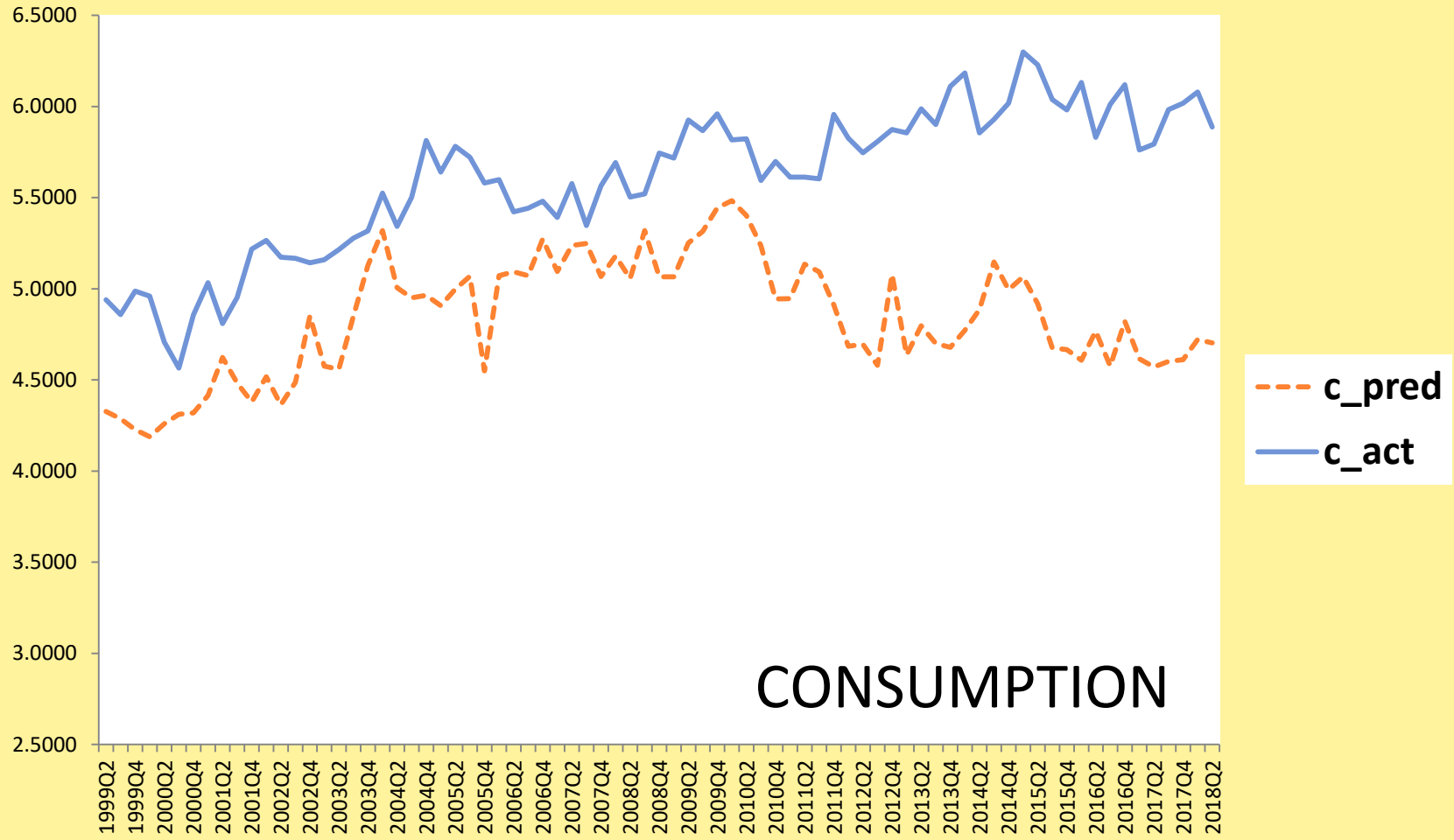
KF predicted Data - GREECE

Calibrated params, predicted unobservables, data for *k* and *h*



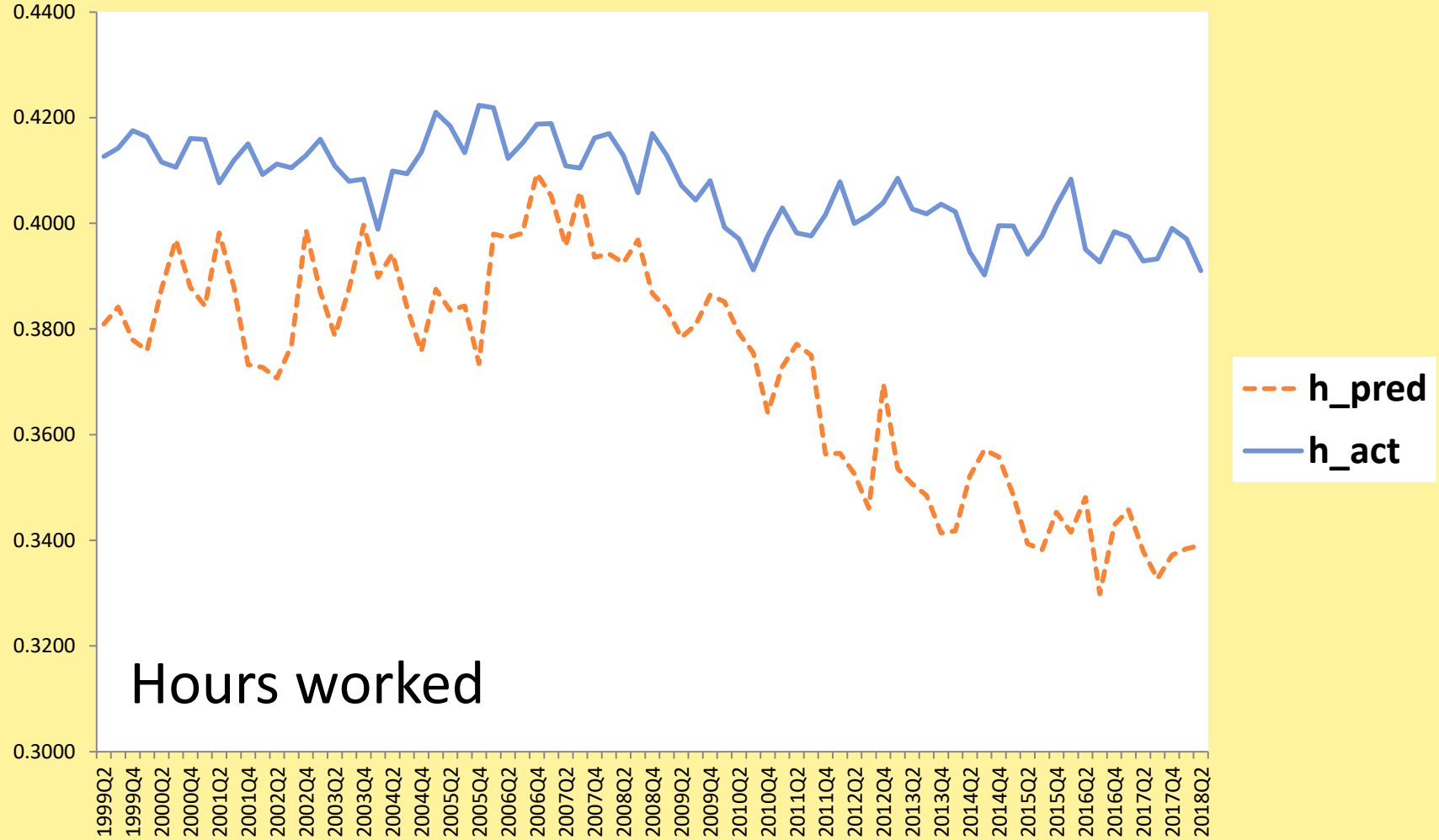
KF predicted Data - GREECE

Calibrated params, predicted unobservables, data for *y, h, TL*



KF predicted Data - GREECE

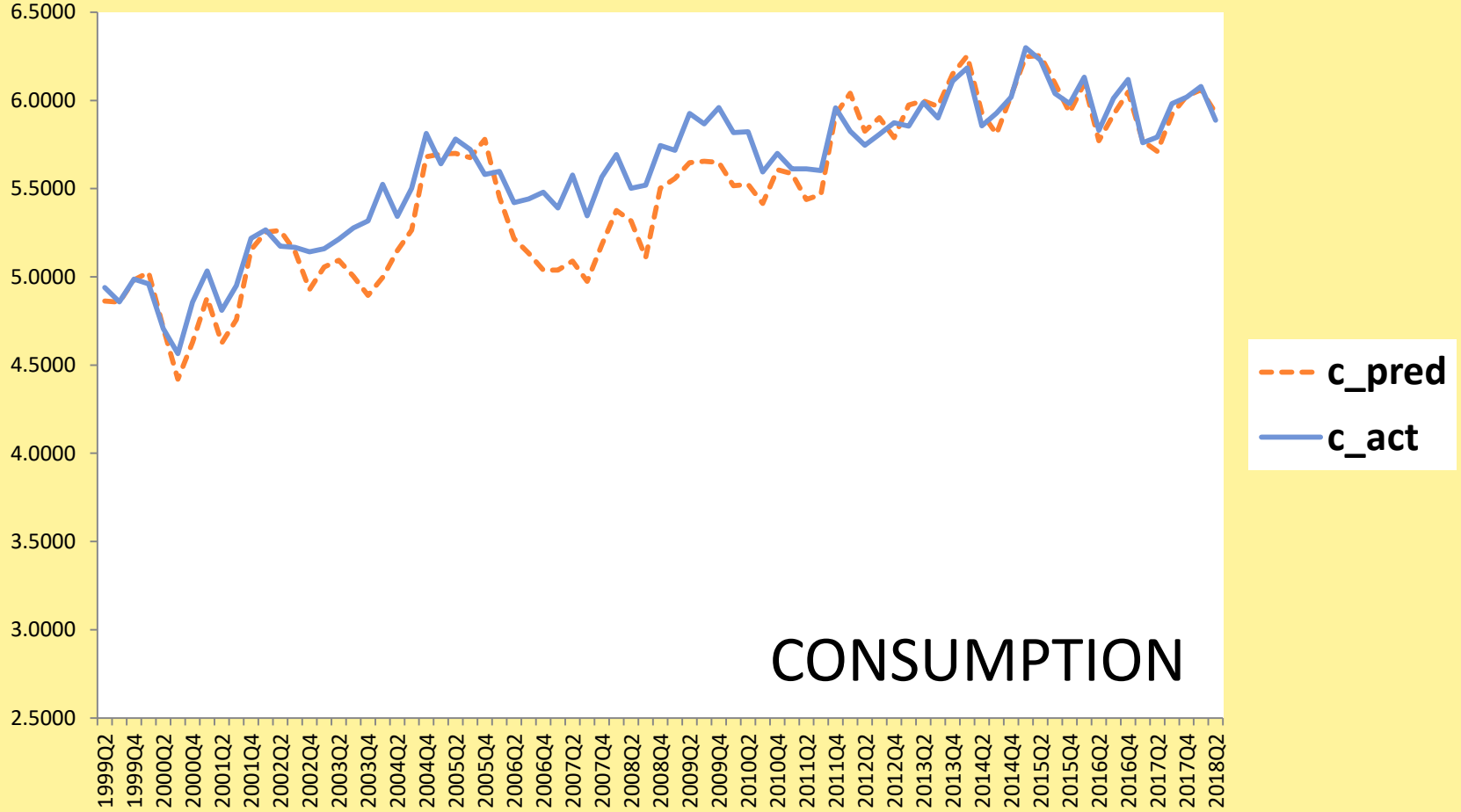
Calibrated params, predicted unobservables, data for *y, c, TL*



KF predicted Data - GREECE

Calibrated params, predicted unobservables, data for *TL*

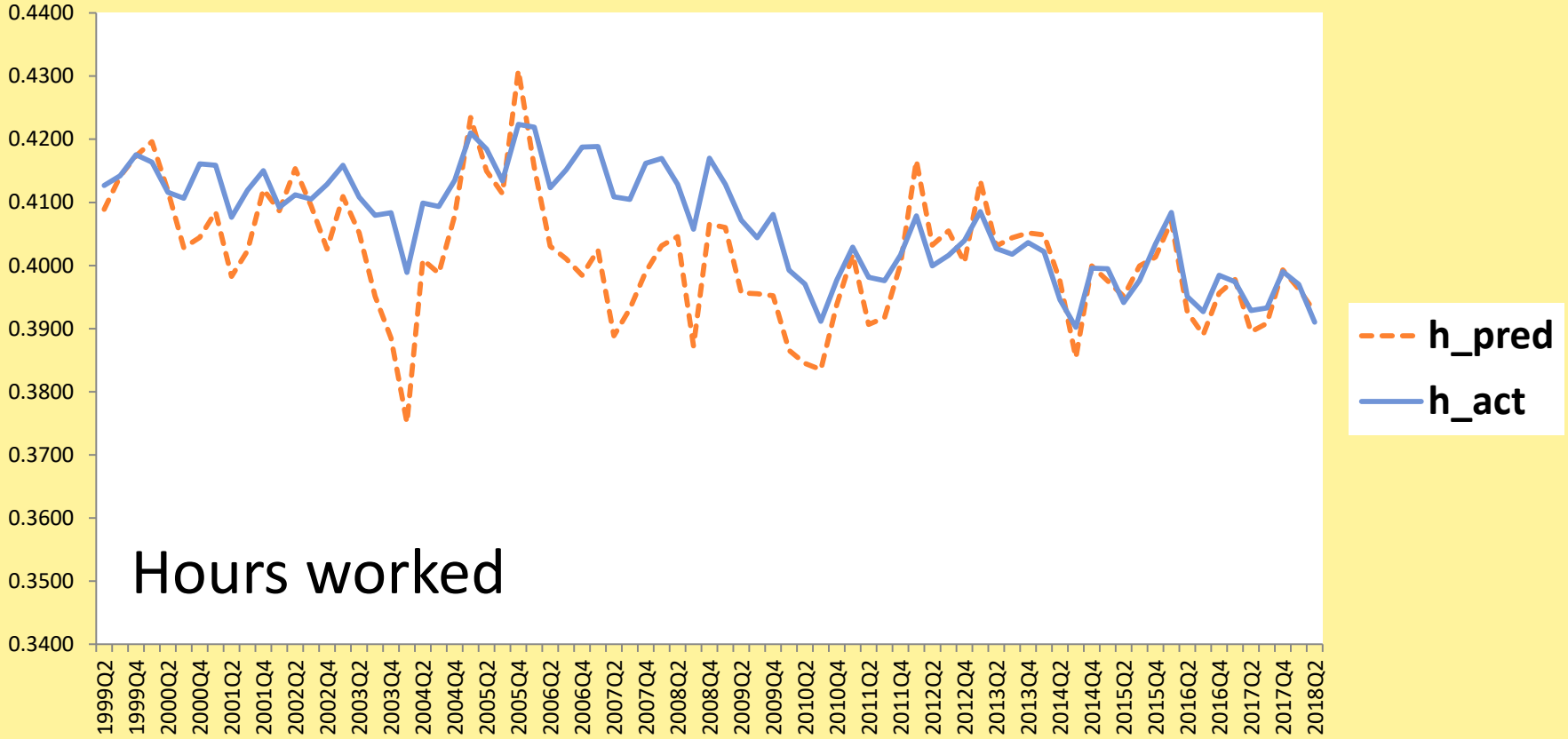
predicted series for y, h



KF predicted Data - GREECE

Calibrated params, predicted unobservables, data for *TL*

predicted series for y, c



Now the bad news

PHASE 2 – Maximum Likelihood

- **FAILED for all countries**
- **Small sample (77 observations)**
- **…and Flat likelihood surface**

PHASE 3 – Markov Chain Monte Carlo

- Metropolis-Hastings Random Walk algorithm (MHRW)

- Most parts of the estimation appear to work acceptably
  Task: work on the Priors to reduce posterior variance
  BUT
- The MHRW does not like the government
- It consistently sends government-related estimands to zero
  And in many cases the Insiders Index also
- This entails a re-thinking of how we model the government empirically

- Note 1: the government is thin on data in the model (only the two effective tax rates participate)
- Note 2: The postulated decomposition of government expenses in "conventional" and "Insiders' generated" is not anchored in anything inside the theoretical PSE model.

PHASE 3 – Government re-engineering

Instead of using the balanced-budget constraint…

$$\bar{\psi}_t + \hat{\psi}\chi_t + \frac{1}{2}\hat{\psi}(\chi_{t+1} - \chi_t)^2 = \tau_t^K \left[\alpha\bar{\omega}_t(\chi_t) - \delta\left(\frac{k_t}{y_t}\right) \right] + (1-\alpha)\tau_t^L \tilde{\omega}_t(\chi_t)$$

…we may argue that the variability of government expenses can be partly explained (not causally) by the level of tax revenues, and by the workings of the Insiders

$$g_t^{data} = \bar{\psi} \cdot \left\{ \tau_t^K \left[\alpha\bar{\omega}_t - \delta\left(\frac{k_t}{y_t}\right) \right] + (1-\alpha)\tau_t^L \tilde{\omega}_t \right\} + \hat{\psi}\chi_t + \frac{1}{2}\tilde{\psi}(\chi_{t+1} - \chi_t)^2$$

- **We bring in more data (government expenses, which we have)**
- **We reduce the number of unobservable states**

Politico-economic equilibrium (PE)

- **The essential difference of the politico-economic equilibrium of the model (PE) , is that the Insiders Index is determined through the maximization of the government's objective function.**
- **Here, the Insiders Index acquires a specific behavioral equation, it will not be modeled as an ad hoc AR(1) series.**
- **Likely, the rest of the structure will remain more or less the same.**

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